Black Scholes equation:

Parabolic PDE that describes the price $V(S,t)$ of an option where S is the price of the asset and t is time:

$$\boxed{\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0}$$

r is annualized risk free interest rate
σ is standard deviation of the stock's returns

$C(S,t)$, the price of a European Call option is a solution with boundary conditions

1. $C(0,t) = 0$ for all t
2. $\lim_{S\to\infty} C(S,t) = S-K$
3. $C(S,T) = \max\{S-K, 0\}$ T is time of expiry

Solution: $C(S_t,t) = N(d_+)S_t - N(d_-)Ke^{-r(T-t)}$ N is cdf of normal dist, N' is pdf of normal dist

- $d_+ = \frac{1}{\sigma\sqrt{T-t}}\left(\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right)$
- $d_- = d_+ - \sigma\sqrt{T-t}$

The corresponding price of a put $P(S_t,t) = Ke^{-r(T-t)} - S_t + C(S_t,t)$

$$= N(-d_-)Ke^{-r(T-t)} - N(-d_+)S_t$$

Black Scholes Assumptions

- $\exists$ risk free interest rate r
- underlying asset follows Geometric Brownian Motion
- No dividend paid
- Can borrow and lend @ rate $= r$
- Can sell any amount of underlying asset
- no arbitrage opportunities
- Frictionless market